[This question paper contains 8 printed pages.]

Your Roll No...............

Sr. No. of Question Paper : 567 G

Unique Paper Code : 2922102302

Name of the Paper : Mathematics for Business Economics – II

Name of the Course : **B.A. (Hons.) Business Economics**

Semester : III

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.

2. Attempt **all** questions. Choice is available within each question.

3. Use of simple calculator is permitted.

P.T.O.

1. Attempt any **four** questions : (4×6=24)

(a) (i) Let $f(x, y) = x^2 + 2xy + y^2$. Find $f(-1, 2)$, $f(a, a)$, and $f(a + h, b) - f(a, b)$.

(ii) Find the domain of the function :

$$\frac{1}{e^{x+y} - 3}$$

(b) (i) Find all first- and second-order partial derivatives for the following :

$$f(x, y) = x^5 \ln y.$$

(ii) Show that $x^2 + y^2 = 6$ is a level curve of

$$f(x, y) = \sqrt{x^2 + y^2} - x^2 - y^2 + 2.$$

(c) (i) Find dz/dt when $z = f(x, y) = xe^{2y}$ with

$x = \sqrt{t}$ and $y = \ln t$.

(ii) Find $\frac{\partial z}{\partial u}$ and $\frac{\partial z}{\partial v}$ if $z = x^3 + y^3$, $x = u^2 - v^2$, $y = u^2 + v^2$.

(d) Let f be a differentiable function of one variable, and let a and b be two constants. Suppose that the equation $x - az = f(y - bz)$ defines z as a differentiable function of x and y. Prove that z satisfies $az'_x + bz'_y = 1$.

(e) (i) Find the partial elasticities of z with respect to t in the following: $z = x^{20} y^{30}$, $x = t + 1$, $y = (t + 1)^2$.

(ii) Find the linear approximation about (0, 0) for the following: $f(x, y) = \sqrt{1 + x + y}$.

2. Attempt any **five** of the following : (5×6=30)

(a) (i) Determine whether the two goods whose demand curves are given below are substitutes or complimentary goods

$$x_1 = \frac{1}{p_1^2 p_2} \text{ and } x_2 = \frac{1}{p_1 p_2}$$

(ii) Examine the concavity/convexity of the following function :

$$g(x,y) = e^{x+y} + e^{x-y} - \frac{3}{2}x - \frac{1}{2}y$$

(b) Find if $Z = \log\left(\frac{\sqrt{x}-\sqrt{y}}{\sqrt{x}+\sqrt{y}}\right)$ is a homogeneous function. Verify Euler's theorem if it is homogeneous.

(c) Show that the production function given by

$$Q = \frac{LK}{L+K}$$

are well behaved? Also calculate their elasticity of substitution.

(d) A monopolist sells his produce in two markets with demand functions given by

$$x_1 = 21 - 0.1p_1 \text{ and } x_2 = 50 - 0.4p_2$$

where x_1 and x_2 are demands in the two markets and p_1 and p_2 are the respective prices The total cost function is given by TC = 10x + 2000. Find the profit maximizing output levels and prices. Check the second order condition.

(e) Examine which of the following functions are homogenous or homothetic or both:

(i) $Z = a \ln x + b \ln y$

(ii) $Z = \ln\left(\frac{x^2}{y^2}\right)$

(iii) $Z = e^{\ln(x^2+xy)}$

(f) What do you understand by convex sets. Show graphically which of the following sets are convex?

(i) $\{\{x,y): x^2 + y^2 > 16\}$

(ii) $\{(x,y): x \geq 0,\ y \geq 0,\ xy \geq 1\}$

3. Attempt any **two** of the following : (2×10=20)

(a) Find all the stationary points of the following function and examine the second order conditions to classify them.

$$f(x,y): x^3 + y^3 - 3xy$$

(b) An individual's utility function for two goods is given by

$$U = (x + 2)(y + 1)$$

It is given that price of x is Rs. 4, price of y is Rs. 6 and the individual's fixed income is Rs. 130. Find the optimum levels of purchase of two commodities that will maximize the individual's utility, check the second order condition.

(c) Let $f(x,y) = x^2 + y^2 + y - 1$, $S = \{(x,y): x^2 + y^2 \leq 1\}$. Find the Extreme points and extreme values for $f(x, y)$ defined over S.

4. Attempt any **two** of the following : (2×8=16)

(a) Find the solution to the following difference equations:

(i) $3x_t = x_{t-1} + 2, \quad x_0 = 2$

(ii) $2x_t + 3x_{t-1} + 2 = 0, \quad x_0 = -1$

(b) Let Y_t denotes national income, I_t total investment and S_t total savings in all period t. Let saving be proportional to national income and investment is proportional to the change in income. Then for t = 1, 2....

$S_t = \alpha Y_t$; $I_t = \beta(Y_t - Y_{t-1})$; $S_t = I_t$ (α and β are positive constants and $\beta > \alpha > 0$). Find the difference equation determining the path of Y_t given Y_0.

(c) Show that $x(t) = C\,e^{-t} + \frac{1}{2}e^{1}$ is the solution of the differential equation $\dot{x}(t) + x(t) = e^{t}$ for all values of C.